

2017

# Complex Adaptive Behavior of Hybrid Teams

Mustafa Canan  
*Old Dominion University*

Andres Sousa-Poza  
*Old Dominion University*

Anthony Dean  
*Old Dominion University*

Follow this and additional works at: [https://digitalcommons.odu.edu/emse\\_fac\\_pubs](https://digitalcommons.odu.edu/emse_fac_pubs)

 Part of the [Operational Research Commons](#), and the [Systems Engineering Commons](#)

## Repository Citation

Canan, Mustafa; Sousa-Poza, Andres; and Dean, Anthony, "Complex Adaptive Behavior of Hybrid Teams" (2017). *Engineering Management & Systems Engineering Faculty Publications*. 27.  
[https://digitalcommons.odu.edu/emse\\_fac\\_pubs/27](https://digitalcommons.odu.edu/emse_fac_pubs/27)

## Original Publication Citation

Canan, M., Sousa-Poza, A., & Dean, A. (2017). Complex adaptive behavior of hybrid teams. *Procedia Computer Science*, 114, 139-148.  
doi:<https://doi.org/10.1016/j.procs.2017.09.013>

This Conference Paper is brought to you for free and open access by the Engineering Management & Systems Engineering at ODU Digital Commons. It has been accepted for inclusion in Engineering Management & Systems Engineering Faculty Publications by an authorized administrator of ODU Digital Commons. For more information, please contact [digitalcommons@odu.edu](mailto:digitalcommons@odu.edu).



Complex Adaptive Systems Conference with Theme: Engineering Cyber Physical Systems, CAS  
October 30 – November 1, 2017, Chicago, Illinois, USA

## Complex Adaptive Behavior of Hybrid Teams

Mustafa Canan<sup>a\*</sup>, Andres Sousa-Poza<sup>a</sup>, Anthony Dean<sup>a</sup>

<sup>a</sup>Engineering Management and Systems Engineering, Old Dominion University, 2101 Engineering Systems Building, Norfolk, VA, 235239

---

### Abstract

The challenges in uncertain, dynamic and complex military operation environments exceed the problem-solving capabilities of individuals. Problem-solving has become a team task. These [hybrid] teams, which typically include machine and human elements, utilize autonomy and artificial intelligence to enhance the quality of actionable information and decision-making capabilities in solving complex problems. For this to be effective, shared mental models must be developed by teams. This demands adaptive behavior of team members to establish a common understanding, and its members to respond to the changes in complex dynamic environments.

In this paper, we introduce a mathematical formalization of an interaction platform designed to support individuals working in heterogeneous, hybrid teams. The purpose of the platform is to facilitate convergent adaptive behavior and interoperability. Hilbert space is used to provide a mathematical foundation and coherent axiomatic structure. Individual and shared mental models are represented in the form of superposition of vector states in a conceptual space. Hilbert Space allows for the inclusion of phenomena, such as spooky activation, entanglement, or emergence that are representative of complex social dynamics.

© 2017 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the scientific committee of the Complex Adaptive Systems Conference with Theme: Engineering Cyber Physical Systems.

**Keywords:** Complex Adaptive Behavior; Hybrid Teams; Agile Teams; Human-Machine Interaction

---

### 1. Introduction

The Greek word “aporia” literally means a dead-end where it is impossible to proceed with a solution. In the context of reasoning, it characterizes any cognitive situation in which a threat of inconsistency confronts the interlocutors. An aporia occurs when a set of perspectives comes in conflict with each other, but are deemed individually plausible. An aporia can emerge in a hybrid team, which is composed of Artificial Intelligent (AI) agents and human agents. A hybrid team acts together to accomplish a given intent. To do so, team members may

---

\* Corresponding author. Tel.: +1-757-683-4558; fax: +1-757-693-5640.

E-mail address: [mcanan@odu.edu](mailto:mcanan@odu.edu)

need to generate new plans, and develop new course of actions to accomplish the given objective. This requires an adaptive behavior of the team members so that problem solving becomes a team task. The perplexities in this type of situation require higher order theories that support the information processing and a self-initiated behavior of the intelligent agents in their interaction with the environment. This means that the interacting agents' actions are constrained to match with the intentions of the members of the hybrid team. The ensuing limitations can impede the adaptive behavior of the actors in the teams.

The self-initiated behavior in AI is constrained with the intent. The intent sets the initial state of the agent and proceeding steps require adaptive behavior to attain the objective. Every attempt to understand the environment is an interaction. In the social paradigm, the interaction of a human between environment and other agents is a generative process, which includes but is not limited to thinking, reasoning, and acquiring knowledge<sup>[1-4]</sup>. The generative process takes place as a continuous interaction with the environment. Due to the uniqueness of every individual, each generative process is unique, and an aporia can emerge<sup>[3, 5, 6]</sup>. An attempt to overcome the inconsistencies will result in adaptive behavior taking place as individuals work to shift the meaning ascribed to a phenomenon<sup>[6]</sup>. The adaptation will either result in a shared phenomenon or continued incongruence. Present approaches oversimplify the nature of the aporia, and as a result, solutions are also oversimplified, or more likely, prone to being wrong or ineffective<sup>[7]</sup>. For example, a method to reduce conflict that is based on the classical prediction of rational choices, will not account for the complexities of the problem. A method built on the assumption that humans behave as classical rational agents will fail to capture the order effects in agent information interaction. Contrary to classical approaches, it has been demonstrated that quantum cognition provides axiomatically coherent answers to demonstrated deviations. The scholarly work in quantum cognition<sup>[8-11]</sup> and Gärdenfor's conceptual space<sup>[12-14]</sup> rendered the Hilbert space geometrical formalism applicable to the known semantic analysis. This formalism and subsequent theoretical developments<sup>[15]</sup> revealed the quantum structure in latent semantic analysis and distributed representation of cognitive structures developed for the purpose of neural networks<sup>[16]</sup>. The effectiveness Hilbert space in information retrieval is recognized by K. Van Rijsbergen<sup>[17]</sup>. The subsequent studies recognized the effects of superposition, uncertainty, and entanglement in the information retrieval<sup>[18]</sup>. D. Widdows demonstrated the effectiveness of quantum logic for connective negation in exploring and analyzing word and meaning<sup>[19, 20]</sup>. Following this development, P. Bruza and his collaborators proved the implication of quantum structures to model semantics space and cognitive structure. They introduced the formalization of context effect about concepts and scrutinized quantum structures of language which include, but are not limited to entanglement, and words in semantic space<sup>[9]</sup>.

A major challenge in modeling the reasoning of a decision-making entity is contextuality. J. Busemeyer introduced the quantum dynamic model of prediction in decision-making<sup>[21]</sup>. In quantum theory, the act of measurement as a contextual action, is known as the observation of a micro level entity. At the macro level, measurement is the act of a decision-making entity. The action alters the state of the agent and the agents' understanding of the environment. This important characteristic improves the complex adaptive behavior of decision-making entities<sup>[11]</sup>. At the same time, A. Khrennikov introduced an advanced quantum decision model<sup>[22, 23]</sup>, which supports the results of Busemeyer's work. The model introduced a complex probability amplitude and demonstrated its efficiency in the algorithm of Prisoners Dilemma and the disjunction effect<sup>[24]</sup>. This research introduced the use of quantum theory outside of the micro world and ameliorated the misconception about the notion of decoherence at the macroscopic level.

The Hilbert space provides a framework to implement the tensor vector algebra. The discussed quantum cognition is not about quantum computing. Rather, it is a method to study the constructs of semantic analysis, conceptual space, and contextuality by using quantum cognitive structures. This can have profound implication in the advancement of AI in adaptive autonomous systems. One main contribution of these developments to AI paradigm is to advance formalization and structuring of artificial knowledge by enhancing it with the introduced quantum mathematical formalism.

The structure of the paper as follows. Section 2 provides a summary of the theoretical foundation of the quantum decision theory such as representing a phenomenon in Hilbert space. In section 2, compatible and incompatible events and the ensuing interference effect in complex situation is discussed with the mathematical formalism of quantum decision theory. Section 3 provides the representation of adaptive behavior of a hybrid system in decision support tool platform. Following this in section 3, a geometric representation of understanding of a shared

phenomenon is exemplified for two interacting agents in a hybrid team. Section 4 is composed summary and conclusion.

## 2. Theoretical Foundation

Mathematical foundation of quantum theory relies on the Hilbert space. Hilbert space is an abstract vector space [2-4]. It is a generalized form of Euclidian space, which can have N dimensions. Hilbert space has several features that make it an ideal foundation on which to develop a modeling approach for complex social phenomena.

### 2.1. Hilbert Space and Vector Notation

Hilbert space consists of abstract points, and each point is called a vector. The state of a system is represented by a vector called "ket"  $|A\rangle$ . A ket in Hilbert space is an object, and it has a dual vector, which is called "bra." A bra is represented as  $\langle A|$ . A ket is represented as a  $N \times 1$  matrix on the specified orthonormal basis.

$$|A\rangle = \mathbf{A} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}, |B\rangle = \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \quad 1$$

A set of vectors  $\{|A_i\rangle\}$ , where  $i = 1, \dots, N$  forms the orthonormal basis for the vector  $|A\rangle$ , which can be expressed as the superposition of the basis vectors:

$$|A\rangle = a_1|A_1\rangle + a_2|A_2\rangle + \dots + a_N|A_N\rangle \quad 2$$

The inner product of two vectors in Hilbert space  $\langle A|B\rangle$  is a complex number. The inner product describes how vector B operates on the vector A in the Hilbert space. The inner product has different meanings. For example if the inner product of two vectors is zero, this means that the vectors are orthogonal.

The events in the Hilbert space are represented by operators. Operators act on vectors as a linear transformation. Operators in Hilbert space are defined by outer products  $|A\rangle\langle B|$ . An outer product maps a vector into a ket:

$$|A\rangle\langle B| \cdot |C\rangle = \langle B|C\rangle \cdot |A\rangle \quad 3$$

Hilbert space contributes to AI studies in two aspects. The first contribution is to represent the human decision making context with basis vectors. In doing so, the quantum probability theories become applicable in the form of projection operators. Secondly, social phenomena can be represented geometrically, which is introduced by Gärdenfors<sup>[12]</sup>. The introduced conceptual model renders geometrical approaches such as pragmatic idealism<sup>[7]</sup> and quantum decision theory<sup>[11]</sup>. Hilbert space formalism is to provide a framework for vector algebra so that the influence of a context can be modeled as a projection operator.

#### 2.1.1. Representing Phenomena in Hilbert Space

Hilbert space is formed by the superposition of the indefinite vector states. Context is represented with a choice of basis. The context is expandable with a basis in Hilbert space. The basis vector for cue, "C" is  $\{x_1, \dots, x_n\}$  can be represented with the superposition of n potential vectors of a phenomenon:

$$|C\rangle = c_1|x_1\rangle + c_2|x_2\rangle + \dots + c_n|x_n\rangle \tag{4}$$

where  $\sum_n |c_n|^2 = 1$  [11]. For simplification consider a word associated with a cue in a context  $c$  can be expressed in a two-dimensional Hilbert space:

$$|W\rangle = a_w|w_c\rangle + a_v|v_c\rangle \tag{5}$$

A vector equation in Hilbert space can also be depicted geometrically, which can be seen in Figure 1.

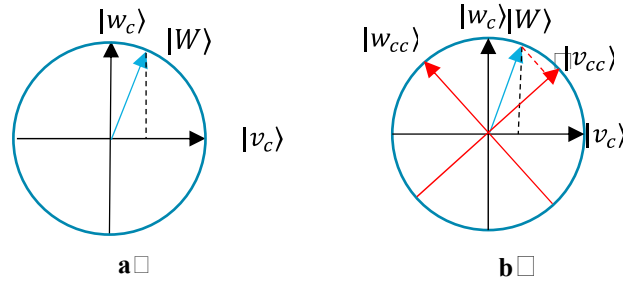


Figure 1 Geometric Representation of Phenomena with basis vectors. The basis vectors represent the context. Same phenomenon can mean different in different context

This geometrical approach renders the mathematical modeling of paradox and ontological subjectivity that give rise uncertainty in complex situations [7].

### 2.2. Quantum Probability

Often, the word quantum generates a perception of particle physics. In fact, the quantum probability theory is a mathematical theory, and from a theory point of view, it is not different than classical probability theory.

In classical probability, events are described by a single sample space. The events in this approach are defined as subsets of a larger set called the sample space. Consider a sample space  $Z$  and two subsets of events  $X$  and  $Y$  are  $X, Y \subseteq Z$ . In the classical approach, the intersection and the union are considered as subset of  $Z$ ,  $(X \cap Y), (X \cup Y) \subseteq Z$ . Consequently, these two are being treated as events in this sample space. There are strict Boolean logic laws that limit the events in classical approaches. For example, the closure axiom and commutative axiom limits the classical probability theory to recognize the paradoxes in the complex situations.

Quantum probability theory introduces a geometric approach to probability. Events are represented as subspaces of vector space, and it allows multiple sample space, which allows both representations of incompatible events and to establish relations between the incompatible event spaces. Quantum probability theory renders to describe the relation among the sample space by unitary transformation operators. The closure property is not required, and events can be non-commutative and non-distributive.

The event space in  $N$ -dimensional Hilbert space is spanned by the orthonormal basis vectors  $\{v = |v_i\rangle, i = 1, \dots, N\}$ . An event  $X$  is spanned by a sub-space in this basis vector space, and the  $X$  corresponds to a projector operator:

$$P_X = \sum_{i \in X} |v_i\rangle \langle v_i| \tag{6}$$

#### 2.2.1. Representation of a System in Hilbert Space

In classical probability theory, a system is always in a definite state. As a result of this, the uncertainty in constructing a system state is constrained and oversimplified. The definite states belong to the sample event space, and the probability function maps the probabilities, which are real numbers, to the elements of the event space.

Quantum probability introduces an indefinite state representation. The state awareness vector is represented by the ket vector  $|A\rangle$ . When a system state is constructed for a situation; the state vector becomes superposition vector in the Hilbert space of all the orthonormal basis. A particular event is represented by a projection operator, and this operator projects the state vector to the event subspace. Suppose the system state vector is:

$$|A\rangle = c_1|v_1\rangle + c_2|v_2\rangle + \dots + c_n|v_n\rangle, \{v = |v_i\rangle, i = 1, \dots, N\} \quad 7$$

In equation 5, the projector operator is introduced for an event  $X$ . The probability of an event  $X$  that spanned by a subspace  $v_x$  is expressed as  $\|P_X|A\rangle\|^2$ .

### 2.3. Adaptive representation of system states

In complex situations, the oversimplification of adaptive behavior of systems generates a threat of reification. To overcome the oversimplification, advanced theories are necessary. Pragmatic idealism, a process philosophy perspective, provides conceptual formalism to complex situations that includes human cognitive dimension<sup>[7]</sup>. Quantum probability theory provides a dynamic mathematical view of the pragmatic idealism of adaptive behavior of systems. In classical probability theory, the cognitive system is a definite state. The measurement is simply recording what existed immediately before the measurement. In quantum probability theory the cognitive systems are in an indefinite state. The measurement is imposing a context to the situation. Measurement creates a definite state, bringing into existence a reality, which was not there before. After the measurement, a new superposition of orthonormal basis is generated. The created superposition can be written as:

$$|A_1\rangle = \frac{P_X|A\rangle}{\|P_X|A\rangle\|} \quad 8$$

### 2.4. Compatible and incompatible events

Simple situations become complex situations with human interventions. In complex situations, uncertainty increases and incompatible events become noticeable. Information becomes accessible from different sources with different views. Characteristics of complex situations introduce constraints that can only be proper formalism. Classical probability theory fails to distinguish the incompatible events. According to the commutative axiom, two events can be realized simultaneously. Quantum probability theory recognizes incompatible events and allows us to establish relations between incompatible events with unitary transformation. As a result of this, in quantum probability, events are processed sequentially and in order of the information matters.

#### 2.4.1. Interference

Quantum probability theory violates the distributive axiom<sup>[11]</sup>, and hence the law of total probability is violated. Violation of total probability is observed in various psychological experiments<sup>[25]</sup>. On the other hand, quantum probability theory provides a coherent explanation to the violations of the classical probability theory. Consider two events  $X, Y$  and these events will be observed subsequently. The probability of the outcome of  $X$  is  $P(Y) = \|P_Y|A\rangle\|^2$ . The probability of  $X$  or not- $X$  ( $\bar{X}$ ) then  $Y$  and the total probability is

$$P_{total} = \|P_Y P_X |A\rangle\|^2 + \|P_Y P_{\bar{X}} |A\rangle\|^2 \quad 9$$

Equation 8 has a salient characteristic that is the inherited state revision (equation 7). The state revision provides a dynamic representation of adaptive behavior. This is important because the occurrence of an event (decision) results in the changes in the system. The state revision is constrained by the sequence of events. Therefore, the shared mental model of a hybrid team must be dynamic in attaining interoperability and agile, adaptive behavior in complex situations. State revision (Equation 7) becomes a contingency in modeling non-compatible events. For example, the projection of event X and Y are  $P_X$  and  $P_Y$ . In the case of being compatible, these projections would give the same probability. However, the non-compatible events do not give the same result. Hence, the state revision should be reflected in the modeling with an effect called interference. Interference is  $Int(XY) = P(Y) - P_{total}(Y)$  which can be derived as:

$$\begin{aligned}
 P(Y) &= \|P_Y|S\rangle\|^2 = \|P_Y I|S\rangle\|^2 \\
 &= \|P_Y(P_X + P_{\bar{X}})|S\rangle\|^2 \\
 &= \langle S|(P_X + P_{\bar{X}})P_Y P_Y(P_X + P_{\bar{X}})|S\rangle \\
 &= \langle S|P_X P_Y P_X|S\rangle + \langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle + \langle S|P_{\bar{X}} P_Y P_{\bar{X}}|S\rangle \\
 &= \|P_Y P_X|S\rangle\|^2 + \|P_Y P_{\bar{X}}|S\rangle\|^2 + [\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle]
 \end{aligned}
 \tag{10}$$

The term  $[\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle]$  corresponds to the interference term. If the interference term is decomposed;

$$\begin{aligned}
 &[\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle] = \\
 &(\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle)(\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle)^*
 \end{aligned}
 \tag{11}$$

$$\begin{aligned}
 &= \langle S|P_{\bar{X}} P_Y P_X|S\rangle \langle S|P_{\bar{X}} P_Y P_X|S\rangle^* + \langle S|P_{\bar{X}} P_Y P_X|S\rangle \langle S|P_X P_Y P_{\bar{X}}|S\rangle^* + \langle S|P_X P_Y P_{\bar{X}}|S\rangle \langle S|P_{\bar{X}} P_Y P_X|S\rangle^* \\
 &\quad + \langle S|P_X P_Y P_{\bar{X}}|S\rangle \langle S|P_X P_Y P_{\bar{X}}|S\rangle^* \\
 &= \langle S|P_{\bar{X}} P_Y P_X|S\rangle \langle S|P_X P_Y P_{\bar{X}}|S\rangle + \langle S|P_{\bar{X}} P_Y P_X|S\rangle \langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle \langle S|P_X P_Y P_{\bar{X}}|S\rangle \\
 &\quad + \langle S|P_X P_Y P_{\bar{X}}|S\rangle \langle S|P_{\bar{X}} P_Y P_X|S\rangle \\
 &= 2 \cdot |\langle S|P_{\bar{X}} P_Y P_X|S\rangle| \cdot \cos \theta
 \end{aligned}
 \tag{12}$$

The interference term can be zero, positive or negative. If the events are compatible events, e.g.  $P_Y P_X = P_X P_Y$ , the order of the event does not matter and interference effect becomes zero. In this case, the total probability is reduced to the classical total probability. In the case of incompatible events,  $P_Y P_X \neq P_X P_Y$ , the prediction will change, and the total law of probability will be violated in the prediction model.

### 3. Adaptive Behavior of Hybrid Teams

Hybrid teams, which typically include machine and human elements, utilize autonomy and artificial intelligence to enhance the quality of actionable information and decision-making capabilities in solving complex problems. For this to be effective, shared mental models must be developed by teams. This demands adaptive behavior of team members to establish the common understanding, as well as adaptive behavior by the team and its members to respond to the changes in complex dynamic environments. The shared mental model will be implemented in a platform that allows communication between the agents of hybrid teams.

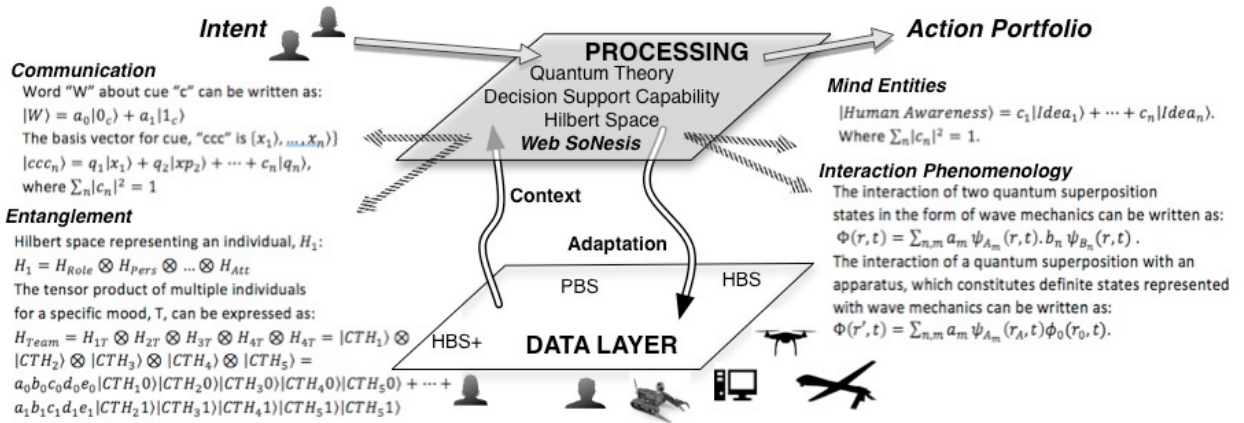


Figure 2 Decision support tool platform for Hybrid teams. PBS is physical based information sources; HBS is human-based information sources (intelligence etc.); HBS+ is the emerging human-based information sources that include the social media tools, world wide web.

A major constraint of a complex joint adaptive behavior is agility. The design of an agile hybrid team requires a dynamic model that includes contextuality. A hybrid team is composed of human members. Thus the quality of actionable information is not only constrained by the computational power of the model but also the comprehensibility. A model based on the quantum probability theory will provide predictions, which include interference of incompatible events and violation of the law of total probability. The platform is based on the Hilbert space representation of phenomena, context, and measurement. A geometrical representation of similarities and difference in this abstract space will be implemented to improve interoperability and develop an agile hybrid team.

### 3.1. Geometric representation of differences in Hilbert space

We demonstrate a simplified interaction example in Hilbert space in which we implement projection and unitary operators. Suppose that two agents of a hybrid team initiate communication from  $A_2$  to  $A_1$  to improve the interoperability. Eventually an adaptive decision making can take place in the complex situation. In this complex situation, the vector  $|C\rangle$  can be projected into two different subspace bases by the agents' projection operators. Agent  $A_1$ 's projection operator is:

$$P_1 = |\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2| \tag{13}$$

when equation 12 operates on the vector  $|C\rangle$ :

$$|C_1\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle. \tag{14}$$

Same calculations for the  $A_2$  generate:

$$P_2 = |\varphi_1\rangle\langle\varphi_1| + |\varphi_2\rangle\langle\varphi_2| \tag{15}$$

$$|C_2\rangle = b_1|\varphi_1\rangle + b_2|\varphi_2\rangle.$$

The interference effect (Equations 10,11, 12) can occur in communication in many different ways. The orders of the locution utterance and the order of actions all constrain the construing process because of the high contextuality.

The effect of communication can be expressed in two ways. By using the transformation operators, one can write the new other than self as:

$$U_{12}A_1 = U_{12}P_1|C\rangle \tag{16}$$

$$U_{12} = |\varphi_1\rangle\langle\varphi_1| + |\varphi_2\rangle\langle\varphi_2|$$



After receiving a stimulus from the agent  $A_2$ , the agent  $A_1$  demonstrate a desire to change the context of the understanding according to the agent  $A_2$ . This can be achieved by a unitary operator that is introduced in equation 16. Unitary transformation in equation 16 describes the relation between domain of awareness of the two agents. Please note that this is an oversimplified example, and time evolution is not expressed in the equation.

$$U_{12}|C_1\rangle = (|\varphi_1\rangle \langle\phi_1| + |\varphi_2\rangle \langle\phi_2|)(a_1|\phi_1\rangle + a_2|\phi_2\rangle) \tag{17}$$

$$= \langle\phi_1|a_1|\phi_1\rangle|\varphi_1\rangle + \langle\phi_1|a_2|\phi_2\rangle|\varphi_1\rangle + \langle\phi_2|a_1|\phi_1\rangle|\varphi_1\rangle + \langle\phi_2|a_2|\phi_2\rangle|\varphi_2\rangle$$

Because of the orthogonality, the second and third terms give zero inner product. Hence:

$$U_{12}P_1|C\rangle = |C_{12}\rangle = \underbrace{a_1}_{\substack{\text{new} \\ \text{amplitude}}} |\varphi_1\rangle + \underbrace{a_2}_{\substack{\text{new} \\ \text{amplitude}}} |\varphi_2\rangle. \tag{18}$$

Even though a transformation occurred as depicted in Figure 3, the projections are still different. This requires an additional projection on  $|C_{12}\rangle$ .

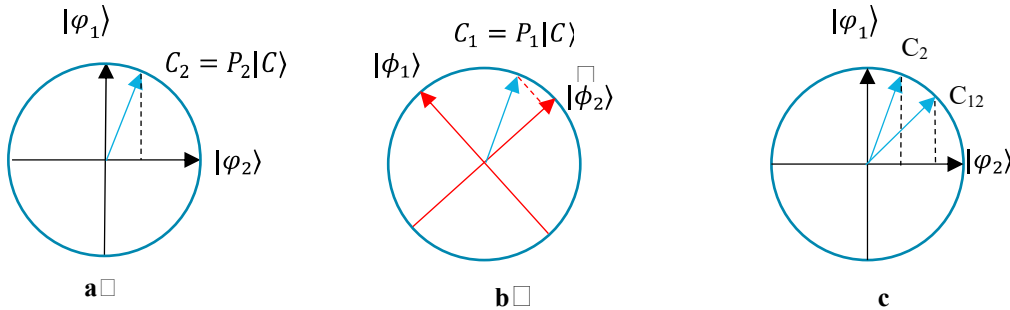


Figure 3 Unitary Transformation of an awareness vector in a complex situation. Figure a represents the projection of the cue vector  $C$  on the context of agent  $A_2$ . Figure b represents the projection of the cue vector  $C$  on the context of agent  $A_1$ .

The difference between two vectors in Figure 3c can be represented as;  $|D\rangle = |C_2\rangle - |C_{12}\rangle$ . 19

the difference between two vectors in Hilbert space can be expressed as:  $d = ||C_2\rangle - |C_{12}\rangle|$  20

By using the inner product, one can have  $d = \frac{(\langle C_2| - \langle C_{12}|, |C_2\rangle - |C_{12}\rangle)}{\sqrt{\langle C_2|C_2\rangle - \langle C_2|C_{12}\rangle - \langle C_{12}|C_2\rangle + \langle C_{12}|C_{12}\rangle}}$  21

Since  $C_{12}$  and  $C_2$  are not orthogonal vectors, the inner product is different than zero. Assume that both vectors are aligned with different basis vectors:

$|C_2\rangle = |\varphi_2\rangle$   
 $|C_{12}\rangle = |\varphi_1\rangle$  22

In this case, the distance between two vectors;

$$d = \sqrt{\frac{\langle C_2|C_2 \rangle}{1} - \frac{\langle C_2|C_{12} \rangle}{0} - \frac{\langle C_{12}|C_2 \rangle}{0} + \frac{\langle C_{12}|C_{12} \rangle}{1}} \quad 23$$

then,

$$d = \sqrt{2}. \quad 24$$

This means two individuals are having a non-degenerate superposition vectors, will have a higher disagreement then:

$$d' = \sqrt{\frac{\langle C_2|C_2 \rangle}{1} - \frac{\langle C_2|C_{12} \rangle}{\neq 0} - \frac{\langle C_{12}|C_2 \rangle}{\neq 0} + \frac{\langle C_{12}|C_{12} \rangle}{1}} \quad 25$$

$$0 < d' < \sqrt{2}. \quad 26$$

This situation can be illustrated as in Figure 3c. The disagreement and agreement become important parameters while maintaining interoperability and attaining adaptive behavior. The geometrical approach introduced provides a platform which not only includes compatible events but also incompatible events. Humans can see the same phenomenon and attain different actions. One reason for this can be projecting the same system state vector into different incompatible events. To recognize projections difference in a shared mental model, it requires establishing a relation between the basis. This can be done Hilbert space by using the quantum probability theories.

#### 4. Conclusion

Classical probability theory has become a limiting factor in studying human behavior. Especially, the concept of rational decision making is confined in a frame conflation of materialism and physicality. This results in exhaustive algorithm or heuristics to model the human behavior with classical probability theory. An alternative and comprehensive approach, quantum decision theory, provides general formalism for the constraints that occur in complex situations because of the incompatible events. Failure to recognize the incompatible events may result in aporia in team decision making. There can be serious consequences; 1) lack of interoperability; 2) information vulnerabilities can impede the operation environment because of the dependencies on the decision aid tools; 3) adaptive behavior can be impeded, because of inadequately available heuristics; 4) agile team decision making can fail to meet the requirements of a complex situation solution domain. To overcome these and similar consequences, a hybrid team should be designed with shared mental models. The comprehensibility of these shared mental models is to be able to establish a relation formalism, which must include time evolution. Quantum decision theory in Hilbert space provides necessary tools to develop comprehensive adaptive behavior in hybrid teams. The design of agile hybrid teams requires a dynamic interaction phenomenology that includes the contextuality and uncertainty. The introduced platform will improve the quality of the actionable information by including real-time information fusion from the indicated sources.

#### References

1. Rescher, N., *Kant and the Reach of Reason*. 1999: Cambridge University Press.
2. Rescher, N., *Process Metaphysics: An Introduction to Process Philosophy* 1996: State University of New York Press.
3. Sousa-Poza, A., *A Narrative of [Complex] Situations and Situations Theory*. 2013. **21**: p. 13-44.
4. Kovacic, S.F., *The Gestalt Imperative: A Proposition for Understanding*. 2013. **21**: p. 95-105.
5. Sousa-Poza, A., Correa Y. *Pragmatic idealism as the basis for understanding complex domains: the trinity and SOSE*. in *IEEE SMC 2005* 2005. Hawaii.
6. Canan, M., A. Sousa-Poza, and S. Kovacic, *Semantic Shift to Pragmatic Meaning in Shared Decision Making: Situation Theory Perspective*. International Journal of Design & Nature and Ecodynamics, 2015. **10**(3): p. 11.
7. Canan, M. and A. Sousa-Poza, *Complex Adaptive Behavior: Pragmatic Idealism*. Procedia Computer Science, 2016. **95**: p. 73-79.
8. Aerts, D., L. Gabora, and S. Sozzo, *Concepts and Their Dynamics-- A Quantum-Theoretic Modeling of Human Thought*. arXiv:1206.1069v2 [cs.AI], 2013.
9. Bruza, P. and R.J. Cole, *Quantum Logic of Semantic Space: An Exploratory Investigation of Context Effects in Practical Reasoning*. arXiv:quant-ph/0612178, 2006.
10. Wendt, A., *Quantum Mind and Social Science: Unifying Physical and Social Ontology*. 2015: Cambridge University Press.
11. Busemeyer, J.R. and P.D. Bruza, *Quantum Models of Cognition and Decision*. 2012: Cambridge University Press.
12. Gardenfors, P., *Conceptual Space: The Geometry of Thought* 2004: MIT Press
13. Aerts, D. and L. Gabora, *A Theory of Concepts and Their Combinations I -- The Structure of the Sets of Contexts and Properties*.

- ArXiv: quant-ph/0402205v1, 2004.
14. Aerts, D. and L. Gabora, *A Theory of Concepts and Their Combinations II -- A Hilbert Space Representation*. arXiv: quant-ph/0402205v1, 2004.
  15. Aerts, D. and M. Czachor, *Quantum Aspects of Semantic Analysis and Symbolic Artificial Intelligence*. J. Phys. A: Math. Gen., 2004. **37**.
  16. Aerts, D., M. Czachor, and S. Sozzo, *Quantum Interaction Approach in Cognition, Artificial Intelligence, and Robotics*. arXiv:1104.3345v1 [cs.AI] 2011.
  17. Rijsbergen, K.J.V., *The Geometry of Information Retrieval* 2004: UK: Cambridge University Press.
  18. Li, Y. and H. Cunningham, *Geometry and Quantum Methods for Information Retrieval*. ACM Sigir Forum, 2008. **42**(2): p. 22-32.
  19. Widdowa, D. *Orthogonal Negation in Vector Spaces for Modeling Word Meaning and Document Retrieval*. in *Proceedings of the 41st Annual Meeting of Association for Computational Linguistics* 2003. Japan.
  20. Widdowa, D. and S. Peters, *Words Vectors and Quantum Logic: Experiments with Negation and Disjunction*. Appeared in *Mathematics of Language*, 2003. **8**: p. 141-154.
  21. Busemeyer, J.R., et al., *A Quantum Theoretical Explanation for Probability Judgment Errors* Psychological Review, 2011. **118**(2): p. 193–218.
  22. Khrennikov, A., *Information Dynamics in Cognitive Psychological Social and Anomalous Phenomena* 2004: Springer Science+Business Media Dordrecht.
  23. Khrennikov, A., *Ubiquitous Quantum Structure*. 2010, London - New York: Springer Heidelberg Dordrecht
  24. Khrennikov, A.Y. and E. Haven, *Quantum mechanics and violations of the sure-thing principle: The use of probability interference and other concepts*. *Journal of Mathematical Psychology*, 2009. **53**(5): p. 378-388.
  25. Tversky, A. and D. Kahneman, *Extensional Versus Intuitive Reasoning The Conjunction Fallacy in Probability Judgement*. *Psychological Review* 1983. **90**(4).